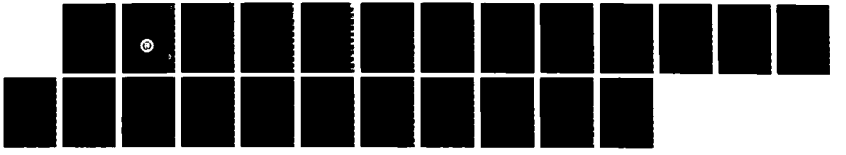
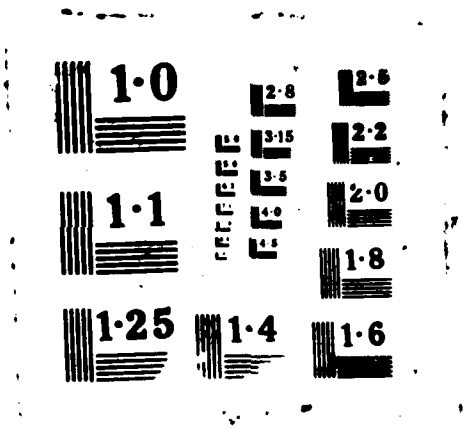
AD-A179 827 GOODNESS OF FIT BASED ON INTEGRATED SQUARED ERRORS IN CHARACTERISTIC FUNC..(U) RENSSELAUER POLYTECHNIC INST TROY NY SCHOOL OF MANAGEMENT A S PAULSON ET AL. 1986 1/1

UNCLASSIFIED WP-37-86-P32 ARO-18072.24-MA F/G 12/1 NL




1·0
1·1
1·25
1·4
1·6
1·8
2·0
2·2
2·5
2·8
3·15
3·5
4·0

ARO 18072.24-MA

DTIC FILE COPY (2)

AD-A179 027

Rensselaer Polytechnic Institute
School of Management

Working Paper No. 37-86-P32

DTIC
ELECTE
APR 03 1987
S D
E

This document has been approved for public release and sale; its distribution is unlimited.

87 4 1 194

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

GOODNESS OF FIT BASED ON INTEGRATED SQUARED ERRORS IN CHARACTERISTIC FUNCTIONS OR DENSITIES

by

A. S. Paulson,
Ho-Ling Hwang, and
J. L. Bryant

Accession For
NTIS GRA&I
DTIC TAB
Unannounced
Justification
By
Distribution/
Availability Codes
Dist
A-1

DTIC COPY INSPECTED 6

School of Management
Rensselaer Polytechnic Institute
Troy, NY 12180-3590
(518)266-6586

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

Note: This paper is not to be quoted without the permission of the author(s). For a list of Working Papers available from the School of Management, please contact Sheila Chao, School of Management.

Goodness of Fit Based on Integrated Squared Errors in Characteristic Functions or Densities

by

A.S. Paulson*

Ho-Ling Hwang**
Rensselaer Polytechnic Institute

and

J.L. Bryant
University of Cincinnati

* Research sponsored in part by the U.S. Army Research Office under contract DAA G29-81-K-0110.

** Now at Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Summary

This paper is concerned mainly with tests of goodness of fit that a random sample $x_1, x_2, \ldots, x_n$ is from a completely specified distribution function. The test is based on the integral of the weighted squared modulus of the difference between sample and population characteristic functions. This integral expression is equivalent to the integral of the squared difference between a density and its Parzen kernel estimate. The asymptotic null distribution of the statistic is that of an infinite weighted sum of mutually independent chi squared variates. An approximation to the asymptotic null distribution is given and applied to give the percentage points of a test of fit of p-variate normality. The test of fit is consistent under mild regularity conditions.

1. Introduction

The characteristic function corresponding to an arbitrary distribution function $F(x)$ is defined by

$$\phi(u) = \int_{-\infty}^{\infty} \exp(iux)dF(x), \tag{1.1}$$

where $i^2 = -1$ and u is a real number. If $x_1, x_2, \ldots, x_n$ is a random sample from $F(x)$, the sample information is completely captured by the sample characteristic function

$$\phi_n(u) = n^{-1} \sum_{j=1}^{n} \exp(iux_j) . \tag{1.2}$$

We are primarily concerned with testing the goodness of fit hypothesis

$$H: \ F(x) = F_0(x) \quad \text{or equivalently} \quad A: \ \phi(u) = \phi_0(u) \tag{1.3}$$

against the general alternative

$$H: \ F(x) \neq F_0(x) \quad \text{or equivalently} \quad A: \ \phi(u) \neq \phi_0(u) .$$

Here $F_0(x)$ is a completely specified distribution function. Most of this paper is devoted to the univariate case. However, the structure of the analysis and the results clearly indicate that many of our results may be extended to the p-variate case with only minor changes. Discussion of the p-variate case is confined to the development of a test for p-variate normality. Tables of percentage points of the statistic and an approximation to the percentage points of the asymptotic distribution of the statistic are also given. The distribution of the test statistic is shown to become asymptotic for very small sample sizes.

The use of characteristic functions in testing hypotheses of fit originated with Heathcote (1972) who suggested that the hypothesis of symmetry of a distribution

$F_0(x)$ could be based on a single value of $\phi_n(u)$, $u \neq 0$. Much subsequent work involving $\phi_n(u)$ as regards tests of fit has centered on the use of one or two u-values; some work is based on $q \geq 2$ u-values. The literature concerning tests of fit based on sample characteristic functions has been recently reviewed and dicussed by Csorgo (1984). The tests for normality of Hall and Welsch (1983) and Csorgo (1986) are based on examining discrepancies between functions of the sample and population characteristic functions over a continuum of u-values. Our tests are constructed by appropriately weighting the modulus squared of the discrepancies ($\phi_n(u)$ - $\phi_U(u)$) and integrating over all real u. These tests are then shown to be equivalent to integrating over the square of the discrepancies between a density and its Parzen kernel estimate.

The following sections provide our main results and sketch the rationale behind the use of sample characteristic functions and density estimators in tests of goodness of fit. Further details and proofs may be found in Bryant and Paulson (1982).

2. Test Statistic and Discussion

A test of the hypothesis H of (1.3) may be effected if $\phi_n(u)$ can be compared against $\phi_0(u)$ in some reasonable way. As a practical consideration based on extensive attempts at development and application, it seems dangerous to make the comparison at only a few points $u_1, u_2, \ldots, u_q$, say, since a characteristic function is not uniquely determined by its value at a finite number of u-values and any resulting test would not be consistent. Furthermore, for large values of $|u|$, $\phi_n(u)$, being mostly noise, contains virtually no information concerning the parent characteristic function $\phi(u)$ of $x_1, x_2, \ldots, x_n$. Because of these considerations we were led to consider

$$I_n = n\Delta_n = n \int_{-\infty}^{\infty} |\phi_n(u) - \phi_0(u)|^2 \, |\gamma(u)|^2 \, du \tag{2.1}$$

$$= n \int_{-\infty}^{\infty} |\omega_n(u) - \omega_0(u)|^2 \, du. \tag{2.2}$$

where $\omega_n(u) = \phi_n(u)\gamma(u)$ and $\omega_0(u) = \phi_0(u)\gamma(u)$.

We shall assume throughout that (1) $\gamma(u)$ is the characteristic function of an absolutely continuous distribution function $F_\gamma(x)$ with corresponding density $f_\gamma(x)$, and (2) that $|\gamma(u)|>0$ for all $u \neq 0$ and that $|\gamma(u)|^2$ is integrable over $(-\infty,\infty)$. Our assumptions concerning $\gamma(u)$ imply (1) that all nonzero discrepancies of $|\phi_n(u) - \phi_0(u)|$ over arbitrary intervals $a<u<b$, $a<b$, provide a positive contribution to the integral in (2.1), and (2) that the integral (2.1) has the possibly more appealing equivalent form (Feller, 1966, Chapter 15, Heathcote, 1977)

$$I_n = 2\pi n \int_{-\infty}^{\infty} (h_n(x) - h_0(x))^2 \, dx, \tag{2.3}$$

where

$$h_n(x) = n^{-1} \sum_{j=1}^{n} f_\gamma(x-x_j) \tag{2.4}$$

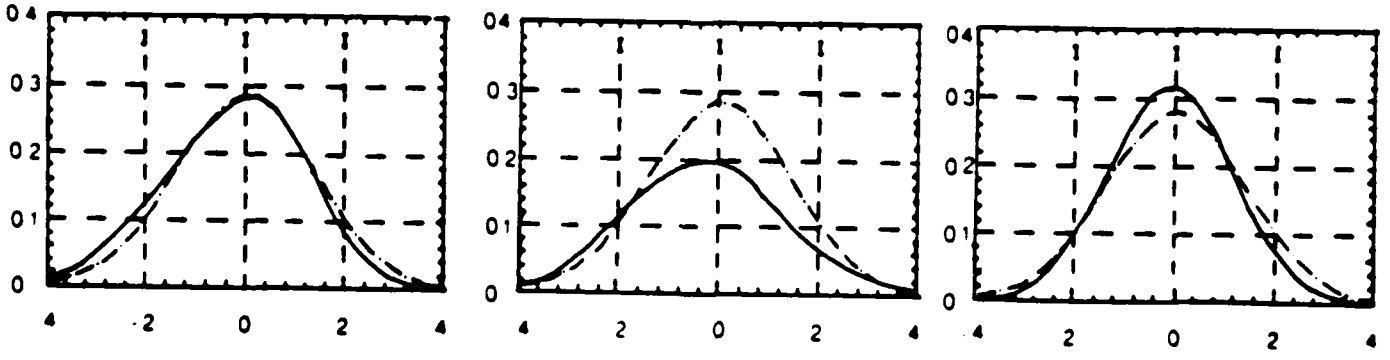
is an unbiased Parzen kernel estimator of $h_0(x)$ if $x_1, x_2, \ldots, x_n$ is a random sample from $f_0(x)$. Since the convolution of an arbitrary distribution with an absolutely continuous distribution is again absolutely continuous (Lukacs, 1970, p. 38), $h_0(x)$ is the density associated with the distribution of $x+z$, where x and z are independent random variables, x has distribution $F_0(x)$ and z has distribution $F_\gamma(x)$. If $F_0(x)$ is absolutely continous with associated density $f_0(x)$, then $h_0(x) = f_0(x)*f_\gamma(x)$, where $*$ signifies the operation of convolution.

As an illustration, we consider the framework for a test of H: the population distribution of $x_1, x_2, \ldots, x_n$ is N(0,1). In this case $\phi_0(u) = \exp(-\frac{1}{2}u^2)$. We choose $\gamma(u) = \exp(-\frac{1}{3}u^2)$ because, conveniently, the convolution of two Gaussian variates is again Gaussian. We thus have

$$h_0(x) = (2\pi^{\frac{1}{2}})^{-1} \exp(-\tfrac{1}{2}x^2), \quad h_n(x) = n^{-1}(2\pi)^{-\frac{1}{2}} \sum_{j=1}^{n} \exp(-\tfrac{1}{2}(x-x_j)^2$$

Some aspects of the character of the test procedure are determinable by small scale simualtion experiments, for example by drawing a random sample $x_1, x_2, \ldots, x_n$ from first a N(0,1) population and subsequently from other populations. Figure 1(a)-(c) depicts $h_0(x)$ for this illustration and typical $h_n(x)$ with n=20 and $x_1, x_2, \ldots, x_n$ a random sample from N(0,1), a random sample from the Cauchy distribution $\pi^{-1}(1+x^2)^{-1}$, and a random sample from the uniform distribution on (-1.14, 1.14) respectively. The parameters of the uniform distribution were determined so that its mean, variance, and skewness matched those of $F_0(x)$. This figure suggests that the test for normality based on I_n will perform well against long-tailed alternatives but less well against symmetric short-tailed alternatives, an observation verified in an unpublished Rensselaer Polytechnic Institute Ph.D. dissertation by Hwang (1984).

Figure 1. Comparison of expected density $h_0(x)$, dashed, with $h_A(x)$, solid, for $h_0(x)$ Gaussian and $h_n(x)$ constructed from (a) Gaussian, (b) Cauchy, and (c) uniform random samples.



Since both $|\phi_n(u) - \phi(u)|^2$ and $|\gamma(u)|^2$ are even functions of u, the integral expression I_n in equivalent to the mathematically more convenient

$$I_n = n \int_{-\infty}^{\infty} |y_n(u) - y(u)|^2 \, |\gamma(u)|^2 \, du \tag{2.5}$$

where the real-valued functions $y(u)$ and $y_n(u)$ are given by

$$y(u) = \mathrm{Re}\phi(u) + \mathrm{Im}\phi(u), \; y_n(u) = \mathrm{Re}\phi_n(u) + \mathrm{Im}\phi_n(u) \; .$$

The function $y(u)$ is also a transform and $y_n(u)$ is its unbiased sample version. There are also data analytic, graphical, and computational advantages to using the transform $y(u)$ instead of $\phi(u)$; for example plots of $y_n(u)$ and $y(u)$ are easier to interpret than separate plots of the real and imaginary parts of $\phi_n(u)$ and $\phi(u)$. Most of the asymptotic results concerning I_n are developed from the covariance kernel $\kappa(u,v)$ of the stochastic process $y_n(u)$,

$$\begin{aligned} K(u,v) &= n \, \mathrm{cov}(y_n(u), \, y_n(v)) \\ &= \mathrm{Re}\phi(u-v) + \mathrm{Im}\phi(u+v) - y(u)y(v). \end{aligned} \tag{2.6}$$

3. Main Results Concerning I_n

We now give the main results concerning I_n of (2.1) and (2.3). Proofs are somewhat lengthy and are not given here; additional details may be found in Bryant and Paulson (1982).

Theorem 1. The goodness of fit statistic $I_n = nA_n$ of equation (2.1) and (2.3) has, when the null hypothesis H is correctly specified, the asymptotic distribution whose characteristic function $c(u)$ is

$$c(u) = \prod_{s=1}^{\infty} (1 - 2\lambda_s iu)^{-\frac{1}{2}}, \tag{3.1}$$

where $\lambda_1, \lambda_2, \ldots$ are the positive eigenvalues of the integral operator K given by

$$Ky(u) = \int_{-\infty}^{\infty} K(u,v)y(v)|\gamma(v)|^2 \, dv, \tag{3.2}$$

where

$$K(u,v) = Re\phi_0(u-v)+Im\phi_0(u+v)-[Re\phi_0(u)+Im\phi_0(u)][Re\phi_0(v)+Im\phi_0(v)].$$

This result characterizes the asymptotic distribution of I_n as that of an infinite weighted sum of independent chi squared variates, each having one degree of freedom, i.e., I_n has the asymptotic distribution of $\sum_{j=1}^{\infty} \lambda_j \chi_j^2$.

Theorem 2. The goodness of fit test of the hypothesis $H: \phi = \phi_0$ based on the statistic I_n of (2.1) is, when the null hypothesis H is correctly specified, consistent.

This result guarantees that the power of the test approaches unity as the sample size increases without bound for any alternate $F \neq F_0$. Many of the tests in the literature based on the use of the chracteristic function do not possess the consistency property of Theorem 2. In particular, the tests of Heathcote (1972), Kellermeier (1980), and Koutrouvelis (1980, 1981), are all based on only a finite number of u-values and therefore cannot be consistent.

Theorem 3. The jth cumulant κ_j of the asymptotic distribution of I_n is

$$\kappa_j = (j-1)!2^{j-1} \sum_{s=1}^{\infty} \lambda_s^j, \quad j \geq 1, \tag{3.3}$$

where

$$\sum_{s=1}^{\infty} \lambda_s^j = \int_{-\infty}^{\infty} K_j(u,u)|\gamma(u)|^2 \, du, \tag{3.4}$$

and the jth iterated kernel $K_j(u,v)$ is defined recursively by $K_1(u,v) = K(u,v)$ and

$$K_j(u,v) = \int_{-\infty}^{\infty} K_{j-1}(u,t)K(t,v)|\gamma(t)|^2 \, dt. \tag{3.5}$$

The cumulant expression (3.5) makes determination of approximations to the limiting distribution of I_n relatively easy to obtain. Furthermore, this expression is easily extended to dimensionality $p > 1$. As it turns out, the Pearson (1959) three cumulant χ^2 approximation produces acceptable results in the upper tail of the distribution of I_n as judged by exact numerical inversion of (3.1) for a variety of special cases. The approximation determined by the weighted sum of two independent chi squared variates on m_1 and m_2 degrees of freedom, $a_1\chi_1^2 + a_2\chi_2^2$, is even better but is a little more difficult to compute (Bryant and Paulson, 1982).

4. Asymptotic Power of the Tests

The developments in this section closely parallel those of Durbin and Knott (1972). Under the null hypothesis we will take the underlying population distribution function $F_0(x)$ to be standard normal with $\phi_0(u) = \exp(-\frac{1}{2}u^2)$ and will consider only simple alternate hypotheses of mean and variance shift. We take $\gamma(u) = \exp(-\frac{1}{2}s^2u^2)$, with s specified as we proceed. Asymptotic power as a measure of performance of a test is not altogether satisfactory since it is liable to be highly dependent on the particular null and alternative distributions chosen for investigation and because of its local character. Nevertheless, the demonstration of adequate

performance in terms of limiting power in some given typical situation lends credibility to the proposed testing procedure. We shall omit the technical details of the development and simply present the results.

Table 1 presents the asymptotic power of the goodness of fit test based on I_n at the 5% level of significance for several values of the scaling parameter m. Identical results were obtained for Gauss-Hermite quadrature of orders 48 and 64, so it is believed that these are good approximations to the asymptotic powers attainable through use of I_n. Also included in Table 1 for comparative purposes are the asymptotic powers of the Cramer-von Mises test, Anderson-Darling test, and Watson's U^2 test, which have been computed by Durbin and Knott (1972).

Note that the value of the scaling factor m strongly influences the behavior of the test. Large m yields tests which are extremely sensitive to mean deviation, since the behavior of the sample characteristic function near the origin dominates the test statistic. At the same time, the performance of the test against variance shift deteriorates. Conversely, smaller values of m will, up to a point, emphasize deviations in variance. If mean and variance shift alternatives are of equal concern, Table 1 indicates that a choice of m between .8 and unity is reasonable.

A comparison of the I_n test (with m = 1) with the other three tests shows that, if mean and variance shifts are of equal concern, the Anderson-Darling test is superior while I_n ranks second. The asymptotic power of I_n in the particular situations addressed here is therefore comparable to those of other commonly accepted procedures. The attractive power properties of I_n in the univariate case indicates that it is worth investigating an extension of I_n to tests of p-variate normality. The tests which obtain from these extensions will be true p-variate tests.

Table 1

Asymptotic Power (in %) of Goodness-of-Fit Tests
Against Mean and Variance Shift in the Two-Sided Situation

		Mean Shift		Variance Shift	
		Best Test - 50%	Best Test - 95%	Best Test - 50%	Best Test = 95%
I_n	m = 4/3	47.4	93.8	12.7	51.0
	m = 1	44.6	92.2	17.6	63.4
	m = 4/5	41.5	90.3	20.2	67.3
	m = 2/3	38.4	88.0	21.1	67.7
Cramer-von Mises		45.7	92.8	8.5	25.8
Anderson-Darling		47.2	93.8	40.1	88.9
Watson's U^2		20.5	60.7	24.9	71.1

5. Test of Fit for p-variate Normality

The test statistic (2.1) as stated need not be affine invariant, a serious drawback in multivariate tests of fit (Cox and Small, 1978). Affine invariance for the Gaussian case is however easily effected. Given a random sample $x_1, x_2, \ldots, x_n$ putatively from $N_p(\mu,D)$, we base a test of fit for $N_p(\mu,D)$ on

$$I_{p,n} = n \int_{R_p} |\psi_n(u)-\psi(u)|^2 \exp(-m^2 u^T u)\, du, \qquad (5.1)$$

where $\psi(u) = \exp(-\frac{1}{2}u^T u)$ and

$$\psi_n(u) = n^{-1} \sum_{j=1}^{n} \exp(iu^T D^{-\frac{1}{2}}(x_j-\mu)). \qquad (5.2)$$

Here $D^{-\frac{1}{2}}$ is the unique inverse of the symmetric square root of the positive definite covariance matrix D and u is a px1 vector of real numbers. The expression (5.1) has the closed form representation

$$I_{p,n} = \pi^{\frac{1}{2}p}\{n^{-1} m^{-p} \sum_j \sum_k \exp(-(2m^2)^{-1}Q_{jk})$$

$$- 2(\tfrac{1}{2}+m^2)^{-\frac{1}{2}p} \sum_j \exp(-(2(1+2m^2))^{-1}Q_j) + n(1+m^2)^{-\frac{1}{2}p}\} \qquad (5.3)$$

where

$$Q_j = (x_j-\mu)^T D^{-1}(x_j-\mu), \quad Q_{jk} = (x_j-x_k)^T(2D)^{-1}(x_j-x_k)\ .$$

A straightforward extension of (3.3) to p-dimensions gives the j-th asymptotic cumulant of $I_{p,n}$ as

$$\kappa_j = (j-1)!2^{j-1} \int_{R_p} K_j(u,u)\exp(-m^2 u^T u)\, du, \qquad (5.4)$$

where the jth iterated kernel $K_j(u,v)$ is defined for $j \geq 2$ by the recursive relationship

$$K_j(u,v) = \int_{R_p} K_{j-1}(u,w)K(w,v)\exp(-m^2w^Tw)\,dw$$

with $K_1(u,v) = K(u,v) = \psi(u-v) - \psi(u)\psi(-v)$, and where u, v and w are px1 vectors of real numbers. By tedious integration, the first three cumulants of the asymptotic distribution of $I_{p,n}$ are determined to be

$$\begin{aligned}
\kappa_1 &= \pi^{\frac{1}{2}p}\{(m^2)^{-\frac{1}{2}p} - (1+m^2)^{-\frac{1}{2}p}\} \\
\kappa_2 &= 2\pi^p\{(m^2(2+m^2))^{-\frac{1}{2}p} - 2((1+m^2)^2-\tfrac{1}{2})^{-\frac{1}{2}p} + (1+m^2)^{-p}\} \qquad (5.5) \\
\kappa_3 &= 8(\pi^3)^{\frac{1}{2}p}(2^p[m^2(3+2m^2)^2]^{-\frac{1}{2}p} - (3)2^{\frac{1}{2}p}[(1+m^2)(1+4m^2+2m^4)]^{-\frac{1}{2}p} \\
&\qquad + (3)2^p[(1+m^2)(1+2m^2)(3+2m^2)]^{-\frac{1}{2}p} - [(1+m^2)^3]^{-\frac{1}{2}p}\}.
\end{aligned}$$

The p dimensional version of Theorem 1 gives the asymptotic null distribution of $I_{p,n}$ as that of

$$\sum_{j=1}^{\infty} \lambda_{pj}\chi_j^2 ,$$

where the χ_j^2 are mutually independent chi squared variates on one degree of freedom and the λ_{pj} are nonnegative weights which satisfy $\Sigma\lambda_{pj} = \kappa_1$.

The variate

$$W = \kappa_1 + \frac{\kappa_3}{4\kappa_2}(\chi_\nu^2 - \nu), \quad \nu = \frac{8\kappa_2^3}{\kappa_3^2}, \qquad (5.6)$$

is constructed to have the same first three cumulants as the asymptotic distribution of

$I_{p,n}$. Accordingly, the upper tail of the asymptotic null distribution of $I_{p,n}$ admits of the approximation (Pearson, 1959)

$$pr(I_{p,n} \leq w) \cong pr(\chi^2_\nu \leq \nu + \frac{4\kappa_2}{\kappa_3}(w-\kappa_1)) \quad . \tag{5.7}$$

The approximate null distribution of $I_{p,n}$ for m=1 and for $1 \leq p \leq 5$ and $p < n \leq 120$ has been developed by simulating 10000 independent realizations of the statistic $I_{p,n}$ for each combination of n and p. The cubic-spline smoothed upper 10, 5, and 1% points are given in Table 2. The entry under n=∞ represents the percentage point computed from (5.7). This table clearly indicates the rapidity of the convergence of the percentage points of $I_{n,p}$ to those of the asymptotic distribution of $I_{n,p}$. Considerable effort was expended in an attempt to find good values of m as a function of p and we finally settled on the choice m=1 as a good one, but it is not optimal in the sense of providing maximal power against any specific alternatives. The situation here involving a choice for m to provide good power against unknown alternatives is similar to that encountered in density estimation: a variety of kernels and window widths give good results for specific situations but no specific choice seems to be uniformly good when only data, and not the parent distribution of the data, are available (Tapia and Thompson, 1978, pp. 24-91 and especially pp. 60-68 and pp. 76-84).

When μ and D are not specified and are estimated from the data by

$$\hat{\mu} = n^{-1}\sum_{j=1}^{n} x_j, \quad \hat{D} = n^{-1}\sum_{j=1}^{n}(x_j-\hat{\mu})(x_j-\hat{\mu})^T \quad ,$$

define

$$\hat{\psi}_n(u) = n^{-1}\sum_{j=1}^{n}\exp(iu^T\hat{D}^{-\frac{1}{2}}(x_j-\hat{\mu})),$$

Table 2. Upper 100α Percent Points for the Statistics $I_{p,n}$ m=1, p=1(1)5, n≥5

		100α		
p	n	10	5	1
	5	1.14	1.51	2.40
	10	1.16	1.55	2.48
1	20	1.18	1.57	2.50
	40	1.18	1.57	2.52
	120	1.19	1.58	2.53
	∞	1.19	1.58	2.53
	5	2.80	3.40	4.76
	10	2.80	3.41	4.78
2	20	2.84	3.44	4.84
	40	2.87	3.48	4.90
	120	2.88	3.50	4.92
	∞	2.89	3.52	4.94
	10	5.70	6.60	8.57
	20	5.72	6.63	8.63
3	40	5.74	6.65	8.68
	120	5.75	6.68	8.73
	∞	5.76	6.70	8.76
	10	10.54	11.89	14.91
	20	10.60	11.95	14.93
4	40	10.64	12.00	14.95
	120	10.69	12.05	14.96
	∞	10.72	12.07	14.97
	10	19.09	20.95	24.91
	20	19.10	20.94	24.75
5	40	19.16	21.03	24.93
	120	19.25	21.17	25.13
	∞	19.31	21.23	25.26

and

$$I_{p,n}(\hat{\mu},\hat{D}) = n \int_{R_p} |\hat{\psi}_n(u) - \psi(u)|^2 \exp(-m^2 u^T u)\, du. \qquad (5.8)$$

The statistic $I_{p,n}(\hat{\mu},\hat{D})$ has the explicit representation (5.3) with $\hat{\mu}$ and $\hat{D}$ substituted for μ and D respectively. The statistic $I_{p,n}(\hat{\mu},\hat{D})$ is affine invariant, is easy to compute, and is based on a complete specification of the p-variate normal which explicitly accounts for all deviations from p-variate normality and thus specifies a true p-variate test. Probabilistic arguments strongly suggest that the goodness of fit test of the composite hypothesis H: the parent distribution of $x_1, x_2, \ldots, x_n$ is $N_p(\mu,D)$, μ and D completely unspecified, based on $I_{p,n}(\hat{\mu}, \hat{D})$ is consistent for all $0<m<\infty$; however, we have not been able to obtain an analytical proof of this. Percentage points for (5.8), $1 \le p \le 10$ are available in Paulson et al. (1986).

Acknowledgement

The work of A.S. Paulson was supported by the U.S. Army Research Office under contract DAAG29-81-K-0110.

References

Bryant, J.L. and Paulson,A.S. (1982). Goodness of fit based on distance between characteristic functions and densities. Operations Research and Statistics Research Report, No. 37-82-P4, Rensselaer Polytechnic Institute.

Cox, D.R. and Small, N.J.H. (1978). Testing multivariate normality. Biometrika, 65, pp. 263-272.

Csorgo, S. (1984). Testing by the empirical characteristic function: A survey. Proceedings of the Third Prague Symposium in Asymptotic Statistics (1983). pp. 45-56. Amsterdam: North-Holland.

Csorgo, S. (1986). Testing for normality in arbitrary dimension. Annals of Statistics, to appear.

Durbin, J. and Knott, M. (1972). Components of Cramer-von Mises Statistics. I Journal of Royal Statistics Society, B, 34, pp. 290-307.

Feller, W. (1966). An Introduction to the Probability Theory and Its Applications, Vol. II. New York: Wiley.

Hall, P., and Welsch, A.H. (1983). A test for normality based on the empirical characteristic function. Biometrika, 70, pp. 485-489.

Heathcote, C.R. (1972). A Test of Goodness of Fit for Symmetric Random Variables, Austral. Journal of Statistics, 14, pp. 172-181.

Heathcote, C.R. (1977). The integrated squared error estimation of parameters. Biometrika, 64, pp. 255-264.

Kellermeier, J. (1980). The empirical characteristic function and large sample hypothesis testing. Journal of Multivariate Analysis, 10, pp. 78-87.

Koutrouvelis, I. (1980). A goodness-of-fit test of simple hypotheses based on the empirical characteristic function. Biometrika, 67, pp. 238-240.

Koutrouvelis, I. (1981). Test for normality in stable laws. Communications in Statistics, A, 10, pp. 581-595.

Lukacs, E. (1970). Characteristic Functions, Darien: Hafner.

Paulson, A.S., Roohan, P., Hwang, H., and Fuller, H. (1986). A test of goodness of fit for multivariate normality. Rensselaer Polytechnic Institute Technical Report 37-85-P31.

Pearson, E.S. (1959). Note on an approximation to the distribution of non-central χ^2. Biometrika, 46, pp. 364.

Tapia, R.A. and Thompson, J.R. (1978). Nonparametric Probability Density Estimation. Baltimore: The Johns Hopkins Press.

AD-A179027

MASTER COPY - FOR REPRODUCTION PURPOSES

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: ARO-12	2. GOVT ACCESSION NO.: N/A
3. RECIPIENT'S CATALOG NUMBER: N/A	
4. TITLE (and Subtitle): Goodness of Fit Based on Integrated Squared Errors in Characteristic Functions or Densities	5. TYPE OF REPORT & PERIOD COVERED: Working Paper
	6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): A.S. Paulson, Ho-Ling Hwang, J.L. Bryant	8. CONTRACT OR GRANT NUMBER(s): DAAG29-81-K-0110
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Rensselaer Polytechnic Institute, Troy, New York 12180	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:
11. CONTROLLING OFFICE NAME AND ADDRESS: U. S. Army Research Office, Post Office Box 12211, Research Triangle Park, NC 27709	12. REPORT DATE:
	13. NUMBER OF PAGES:
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

multivariate normality, parametric density estimation, power, generalized distances, simulation, asymptotic distribution, covariance kernel, completely specified null, characteristic function

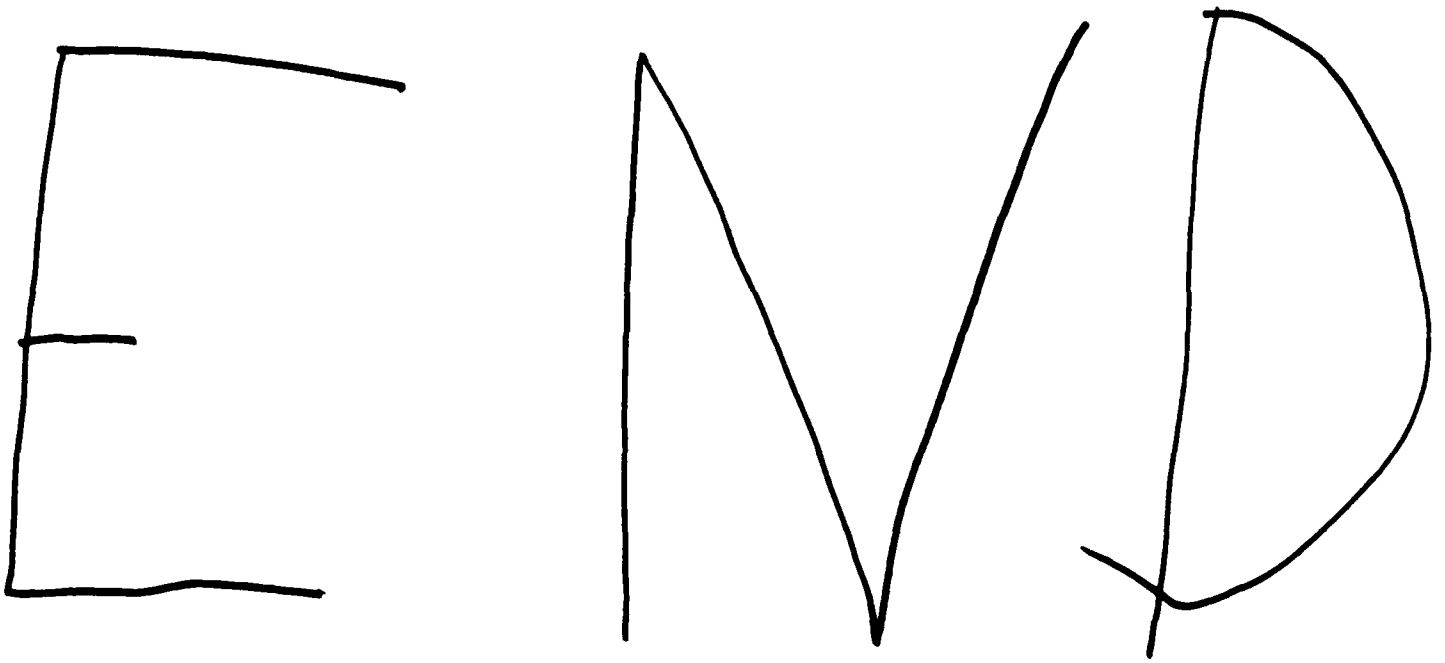
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

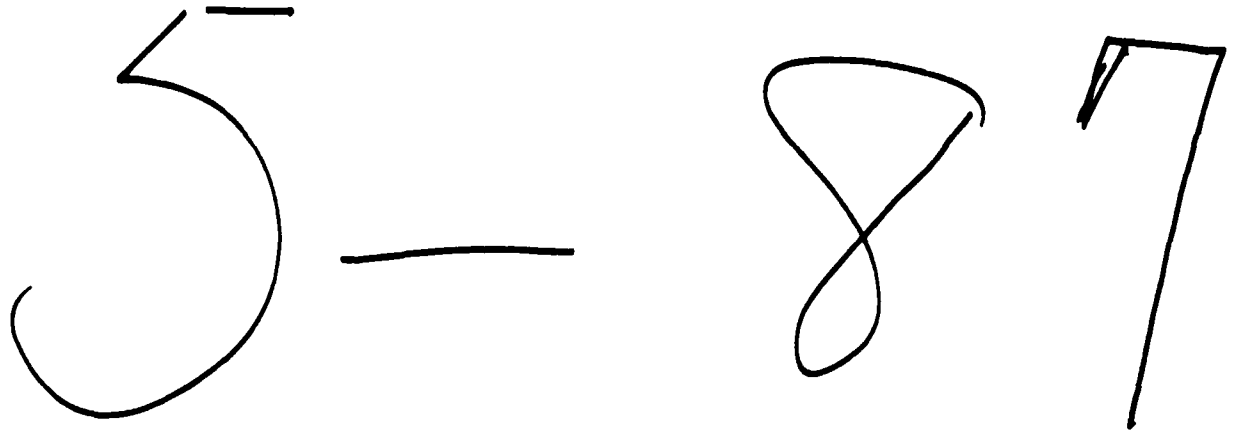
This paper is concerned mainly with tests of goodness of fit that a random sample $x_1, x_2, \ldots, x_n$ is from a completely specified distribution function. The test is based on the integral of the weighted squared modulus of the difference between sample and population characteristic functions. This integral expression is equivalent to the integral of the squared difference between a density and its Parzen kernel estimate. The asymptotic null distribution of the statistic is that of an infinite weighted sum of mutually independent chi squared variates. An

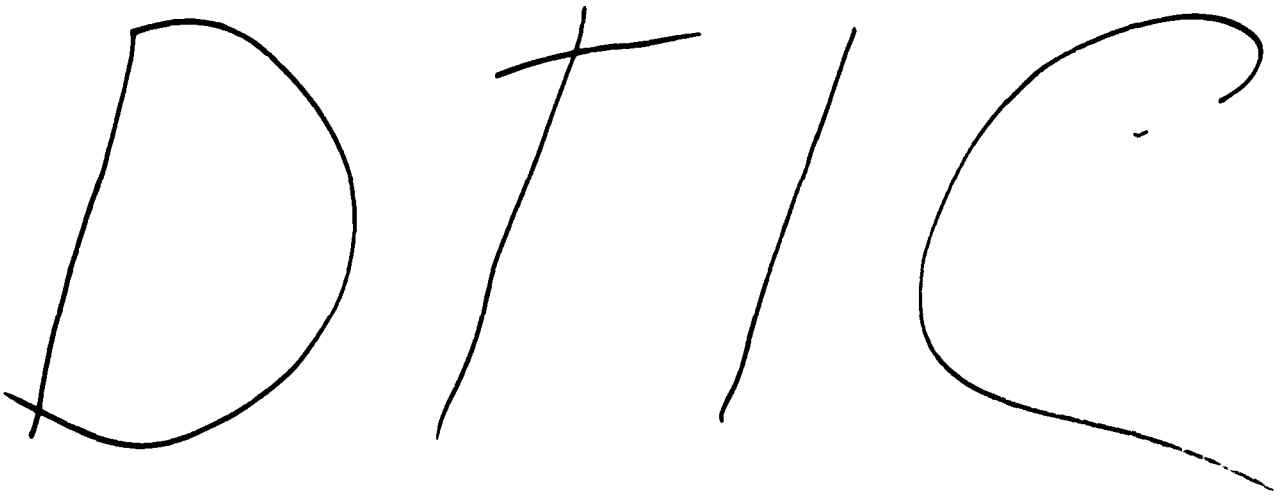
DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

20. approximation to the asymptotic null distribution is given and applied to give the percentage points of a test of fit of p-variate normality. The test of fit is consistent under mild regularity conditions.


E N D


5 - 87


D T I C